Final Project
MULTI CHANNEL 64-QAM ANALYSIS

Giacomo Calabria
Advanced Fiber Optical Communication
Universitat Politècnica de Catalunya (UPC)
3 June 2024

CONTENTS

Abstract**—we investigate the effects of chromatic dispersion and multi-channel multiplexing interference in a multi-channel 64-QAM optical communication system. Utilising MATLAB simulations, we analyse the propagation mechanisms and present various plots to assess channel behaviour. We also provide a detailed overview of the model and mathematical equations of the components used in the project.**

I. INTRODUCTION

In this final project, the effects of chromatic dispersion and multi-channel multiplexing interference will be investigated. A complete communication system will be developed, starting with the generation of PRBS sequences up to the complete decoding of a bit sequence at the receiver. The process of encoding involves the transformation of raw data (bit streams) into a format that is suitable for transmission medium. Conversely, the process of decoding reverses this process, with the aim of retrieving the original information at the receiving end.

We will apply the propagation mechanisms we know from the literature and performed simulations on what developed with MATLAB. We have chosen to report some plots of the constellation during different phases of the propagation. Furthermore, more complex tools such as eye diagram were used to analyse the channel behaviour.

Firstly we illustrate the model and mathematical equations of the various components used in the project. Then we will illustrate how we perform our simulations and then the obtained results.

A. *Optical channel*

The propagation of a pulse in an optical fiber can be described by a fundamental equation known as the Nonlinear Schrodinger Equation (NLSE):

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A + \beta_1 \frac{\partial A}{\partial t} + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3} - j\gamma|A|^2 A \quad (1)$$

The NLSE consider three key phenomena that influence the pulse behaviour:

1) Attenuation;
2) Dispersion (Delay, Chromatic, Slope);
3) Non linearities.

In the project, it was decided to include only the effects of attenuation and Chromatic Dispersion.

1) Attenuation: one of the main limiting factors in light propagation is optical loss as it reduces the average optical power reaching the receiver. Under general conditions of power attenuation inside an optical fiber, the attenuation coefficient (that counts different origins of attenuation) α of the optical power P can be expressed as:

$$\frac{dP}{dz} = -\alpha P \tag{2}$$

Hence, considering an optical signal with an optical power P_{in} entering in a fiber of length L the the optical power output P_{out} is expressed as:

$$P_{out} = P_{in} e^{-\alpha L} \tag{3}$$

In Fig. 1 we can see how attenuation affects on the propagation of a light pulse.

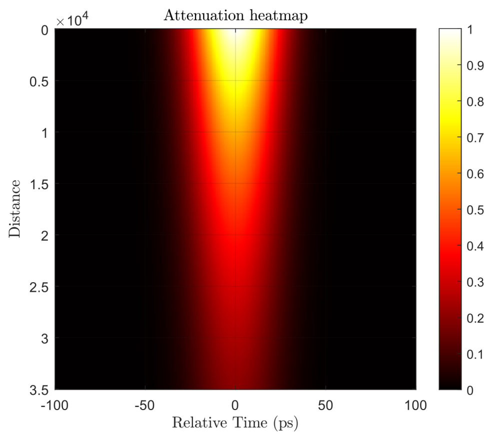

Fig. 1. heatmap of the attenuation of a pulse

2) Pulse Chromatic Dispersion: takes origins from Group Velocity Dispersion (GVD) i.e. the frequency dependence of fiber's refractive index, causing the group velocity dispersion and on the receiver side the broadening of the pulse. This effect is described by the dispersion coefficient β_2 or by the dispersion parameter D. Mathematically, we can describe the output field as

$$\mathcal{A}(z,\omega) = \mathcal{A}(0,\omega) e^{-j\omega^2 z(\beta_2/2)} \tag{4}$$

In Fig. 2 we can see how chromatic dispersion affects on the propagation broadening of a light pulse.

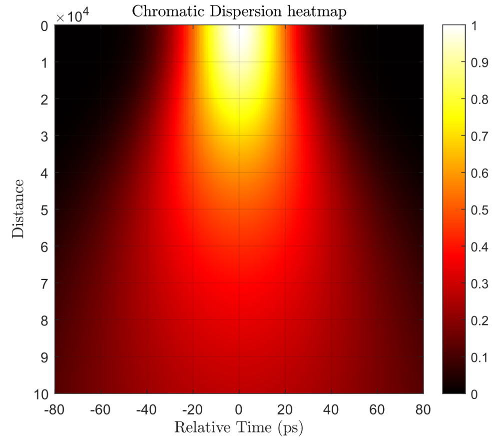

Fig. 2. heatmap of the chromatic dispersion effect on a pulse

B. 64-QAM scheme

Quadrature Amplitude Modulation (QAM) is a digital modulation methods widely used. It conveys two digital bit streams, the I and Q signals, by changing the amplitudes of two orthogonal carrier waves. Based on the numbers of levels in the scheme we can have multiple points in the constellation. In Fig. 3 we can see the typical constellation for a QAM scheme.

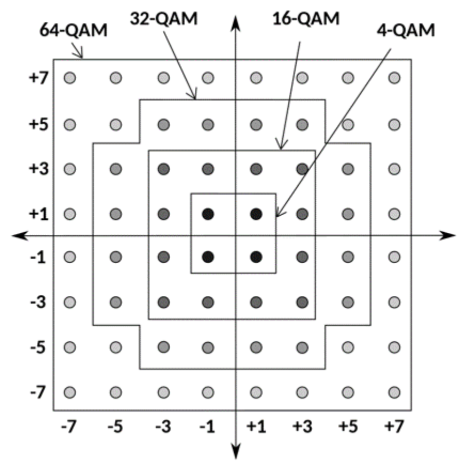

Fig. 3. Constellation points for 4-QAM, 16-QAM, 32-QAM, and 64-QAM overlapped. From [8]

C. Modulator

Optical modulators play a crucial role in fiber-optic communication systems by enabling the encoding of information onto a lightwave carrier. This process, known as modulation, allows for the transmission of digital data or analog signals over optical fibers.

In **external modulation**, a continuous laser generates a steady optical wave, which is then modulated by a separate device. This method allows for greater control and flexibility, enabling manipulation of the light's intensity, phase, or polarisation. External modulation maintains the laser's spectral purity, resulting in better signal quality and allowing for the selection of the optimal modulation technique based on specific needs.

1) Mach-Zehnder modulator: the simplest external modulator and it is used to control light intensity with an electric signal. The output optical field of a MZ modulator is:

$$E_o(t) = E_i\alpha_M \cos\left(\frac{\Delta\phi(t)}{2}\right) e^{j\frac{\Delta\phi(t)}{2}} \tag{5}$$

where

- α_M is the modulator total insertion loss (assumed 1).
- $E_i = E_0 e^{j\omega t}$ is the input field (from the laser diode)
- $\Delta\phi(t)$ is the time-dependent phase of the signal applied.
- V_π is the driving voltage, such as a π phase shift is exerted on the light wave carrier along the optical math of the MZM.

It is also useful to define

$$\phi_U(t) = \pi\frac{V_U(t)}{V_\pi} \quad \phi_L(t) = \pi\frac{V_L(t)}{V_\pi} \tag{6}$$

hence

$$\Delta\phi(t) = \phi_U(t) - \phi_L(t) \tag{7}$$

2) I-Q modulator: this device can manipulate the intensity and phase of an optical carrier to encode information. It splits a base band optical signal, using two MZMs it modulates the in-phase and quadrature components of the carrier before recombination. An scheme of it is represented in Fig. 4

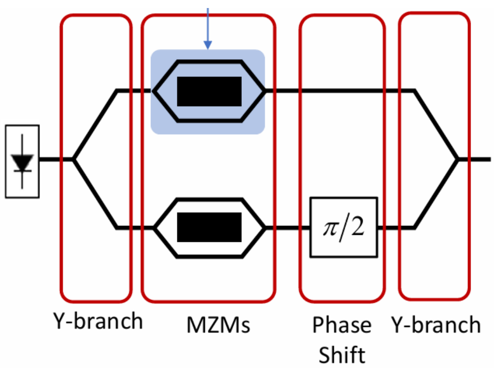

Fig. 4. I-Q modulator. From [4]

The transfer function is:

$$E_o^{I-QM} = \frac{1}{2}\alpha_M \left\{\cos\left(\pi\frac{V_I(t)}{2V_\pi}\right) + j\cos\left(\pi\frac{V_Q(t)}{2V_\pi}\right)\right\} E_i \tag{8}$$

D. Detection

On the other side, an *optical detector* is a device that converts light signals into electrical signals. In **coherent detection** the receivers detects the optical signal by mixing optical fields with that of a local oscillator, a higher amplitude laser of the same or different frequency so that their beating product would result in demodulated signals preserving the signals in both phase and amplitude characteristics in the electronic domain.

1) Phase-diversity coherent receiver: this scheme of an coherent receiver that is able to detect both I and Q signal. The scheme is represented in Fig.5

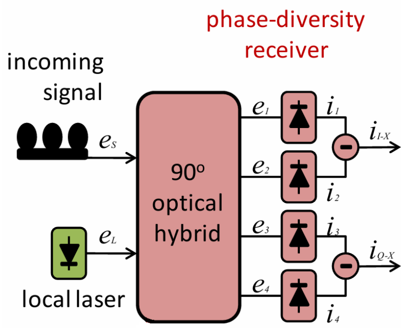

Fig. 5. Phase diversity coherent receiver. From [5]

The output signals of an P-D coherent receiver are:

$$i_{\text{I-X}}(t) = R\,|A_S(t)|\,|A_L|\cos\left(\omega_{FI}t + \theta(t)\right) \tag{9}$$

$$i_{\text{Q-X}}(t) = R\,|A_S(t)|\,|A_L|\sin\left(\omega_{FI}t + \theta(t)\right) \tag{10}$$

Where A_L is the local oscillator laser amplitude and R is the responsivity of the receiver module. These two signals at the end provide all the information transmitted.

E. Wavelength Division Multiplexing

Wavelength Division Multiplexing (WDM) is a fiber optic transmission technique that enables the use of multiple light wavelengths to send data over the same medium, enabling to enhance capacity transmission for high bandwidth.
Due to the physical properties of light, all the wavelengths are independent since the channels don't interfere with each other. However, when separating optical signals, the optical filter doesn't guarantee complete channel idealist, causing interference between channels. Hence, propagation dispersion can degrade the quality and the performance of the overall system.

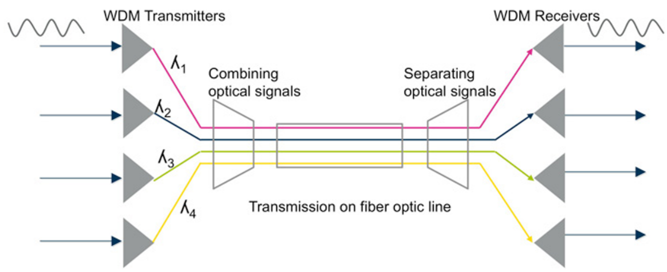

Fig. 6. WDM principle scheme. From [7]

F. Pseudo Random Bit Sequence

Pseudo Random Bit Sequence (PRBS) are sequence of bits that appears random but they are generated by a deterministic algorithm that exhibit statistical properties like truly random sequences. We define the **order** n of a PRBS as the length of the shift register used in generating the sequence. The order determines the period of the sequence, which is $2^n - 1$ bits long before it repeats [6]. High order PRBSs provide longer sequences and more complexity, enhancing their effectiveness in mimicking true randomness over long periods.

II. Simulation

The step of the simulation can be resumed in this way:

1) Generate a PRBS sequence
2) S/P converter with 64-QAM levels
3) I-Q modulator for the three channels
4) Multiplex WDM the three channels
   Attenuation effects
   Chromatic Dispersion effects
5) Demultiplex WDM with optical filtering
6) P-D coherent receiver for the three channels
7) P/S converter with 64-QAM levels
8) BER measurement comparing bit sequences

In Fig. 7 we have shown a general scheme of the simulation system that we have studied.

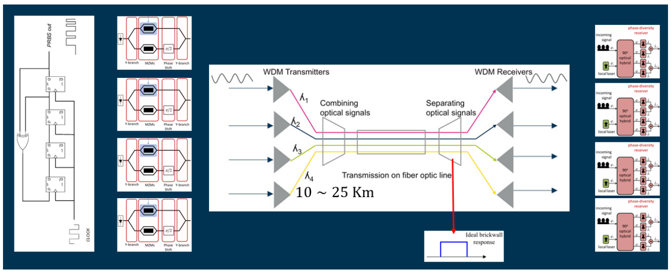

Fig. 7. Summary of the simulation systems

Here are listed the parameters of the MATLAB simulations:

- Fiber length: $L = 10, 15, 25$ Km.
- Attenuation: $\alpha_{dB} = 0.2$ dB/Km.
- Dispersion parameter: $D = 17$ ps/nm/Km.
- Wavelength: $\lambda = 1550$ nm.
- Laser input power: $P_{in} = 1$ mW
- Receiver reponsivity: $R = .95$
- WDM channels: $\omega_0 \pm 50$ GHz.
- WDM filter width: 50 GHz.
- Pulse frequency: 10 GHz.
- Number of symbols: 2^8.
- Samples per symbols: 2^{10}.
- Modulation scheme: 64-QAM.
- Driving voltage: $V_\pi = 4$ V.
- PRBS order: 32.

III. Modulation

After generating a PRBS bit sequence, we have used an I-Q modulator, controlled by an 64-QAM modulation, to generate three optical signals. Then these tree optical signals were multiplexed by a superposition, to obtain a single optical signal.

A. I and Q signals

A point in the constellation of a 64-QAM can be represented with 6 bits, 3 for the I-part and 3 for the Q-part. We have implemented a Serial-to-Parallel converter to group 3 bits from the bit string, generated by the PRBS, to obtain a sequence of I and Q levels of the 64-QAM scheme.

B. Pre-distortion

To ensure efficient utilisation of the I-Q modulator's dynamic range and prevent symbol constellation distortion, we implemented a predistortion technique on the 64-QAM signal. This predistortion compensates for non-linearities in the I-Q modulator, allowing the constellation points to achieve their intended spacing and amplitude distribution.

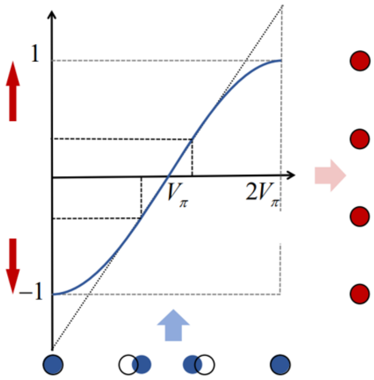

Fig. 8. Pre-distortion principle. From [4]

To achieve the desired modulation levels of the 64-QAM, we employed an inverse cosine $\arccos()$ function in the electrical domain, in order to obtain the proper levels in the optical domain.

C. I-Q modulator

From the sequence of I and Q levels of the 64-QAM modulation generated electrical signals using a rectangular pulse as the ideal base pulse, and using a Gaussian filter we provide some rise/fall times. Subsequently, the two signals were adapted to the threshold voltage scale V_π of the I-Q modulator, and an offset of $-2V_\pi$ was applied to prevent signal inversion. Then, the I-Q modulator were implemented as described by (8).

IV. Optical Channel

After having obtained the multiplexed signal from the three modulators, we applied the effects of attenuation and Chromatic Dispersion to the single signal.

A. Attenuation

The effect of attenuation was applied by follows what is dictates (2). The effect of attenuation in the absence of noise at the receiver/amplifier is not relevant.

B. Chromatic Dispersion

The effect of Chromatic Dispersion was applied by follows what is dictates (4). Since chromatic dispersion and attenuation are two independent effects, it was possible to apply first one and then the other without giving up any combined effect. To compensate optically the effect of chromatic dispersion, another piece of optical fibre with opposite D coefficient and of the same length can be added.

C. Optical filtering

In order to separate the three multiplexed channels using the WDM technique, an optical filter had to be implemented. The choice was placed on an ideal rectangular filter, the operation of which can be found in Fig. 9.

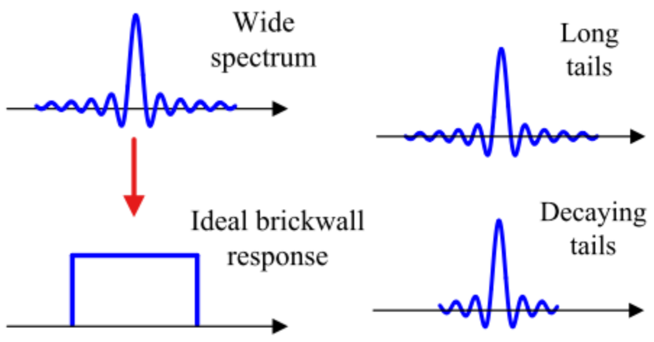

Fig. 9. Ideal rectangular filter. From [9]

The filter width was maximised to the same size as the separation of the three channels (50 GHz). Thanks to the filter operation, we are able to separate the three channels.

V. Detection

On the receiver side the three separated optical channels were converted into the electrical signals by using a Phase-Diversity Coherent Receiver. In order to obtain a bit sequence firstly a simple demodulating scheme has been used and then a Parallel-to-Serial converter has finally used.

A. Phase-Diversity Coherent Receiver

We have develop an Phase-Diversity Coherent Receiver, design as depicted in Fig. 5, one for each channels. We configured the responsivity to $R = 0.95$ and the local oscillator (LO) laser power was set to $P_{in} = 1$ mW. Due to the WDM interference and CD it was chosen to take the average of 2^6 samples around each sampling point.

B. Demobulate bit sequence

After obtained a sample from the receiver the levels were decoded by repositioning the points in the 64-QAM constellation. The, using a Parallel-to-Serial converter, a bit string was finally regenerated by which the error rate could be calculated by comparing the transmitted/received bits string.

VI. Results

In this section we provide the result that we have obtained with the modelled system. We will investigate the results starting from the behaviour of the I-Q modulator, then how the WDM works and the distortions introduced by the WDM and the CD by using the eye diagram and the constellations. Finally, we take care about the compensation of the Chromatic Dispersion in the optical channel and the effects of different PRBS order in the evaluation of the results.

A. Pre-distortion

In Fig. 10 we have reported the obtained constellation of the 64-QAM, as we can see it present a regular structure as we expected after applying a the pre-distortion at the electric signal of the I-Q modulator.

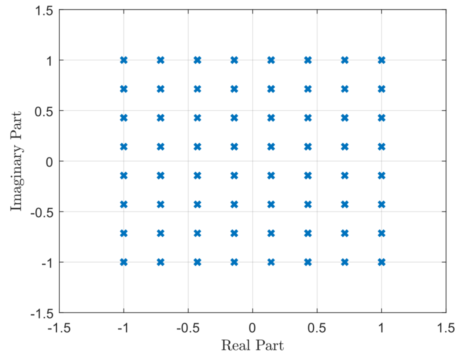

Fig. 10. Constellation of the 64-QAM

This regular structure implying equal spacing between symbol points. This characteristic indicates that the employed 64-QAM scheme effectively utilises the full swing range of the I-Q modulator. We can appreciate how the pre-distortion is applied to the electrical signal in Fig. 11 by observing the main levels of the electrical signal.

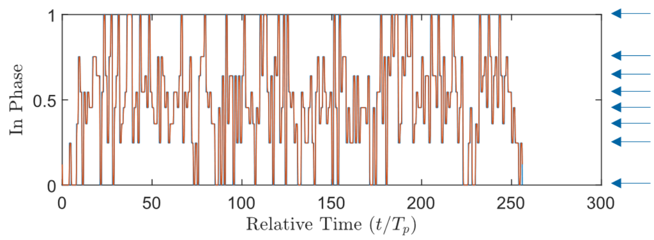

Fig. 11. Signal's levels from the 64-QAM with the pre-distortion applied.

B. WDM

At the basis of WDM technique there is the superposition of the three optical channels, that they are separated by a displacement of 50 GHz. We can analyse the spectrum of the combined signal in Fig. 12

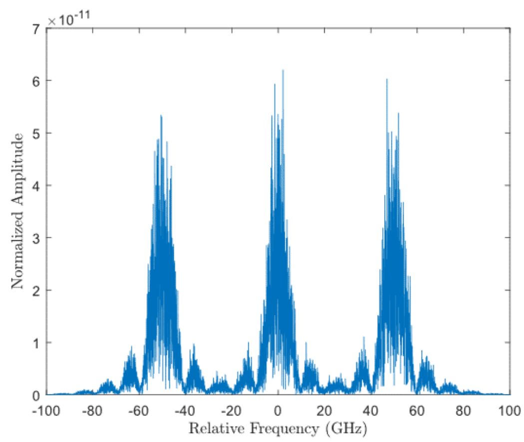

Fig. 12. Spectrum of the multiplexed signal

The three channels are well separated in the frequency domain. We have also noticed that the spectrum is not altered by the propagation process. Using the ideal rectangular filter explained in section ... we have separate the three multiplexed channels. In Fig. 13 we have reported the spectrum of the third channel ($\omega_0 + 50$ GHz) after applying the filter and we can easily see that the spectrum is cut outside the filter window.

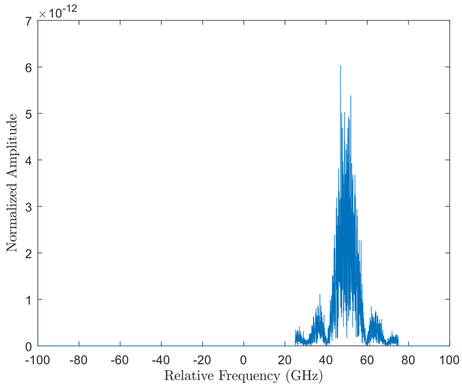

Fig. 13. Filtered spectre of the channel $\omega_0 + 50$ GHz

Even if ideally the WDM technique is completely free of interference and therefore the three channels appear independent to the receiver, the nature of the filter does not allow perfect separation of the three channels, this is reflected in an frequency interference between the three channels. We can appreciate this interference in the Fig. 14. That signal comes from a 16-QAM scheme in order to better evaluate the distortion.

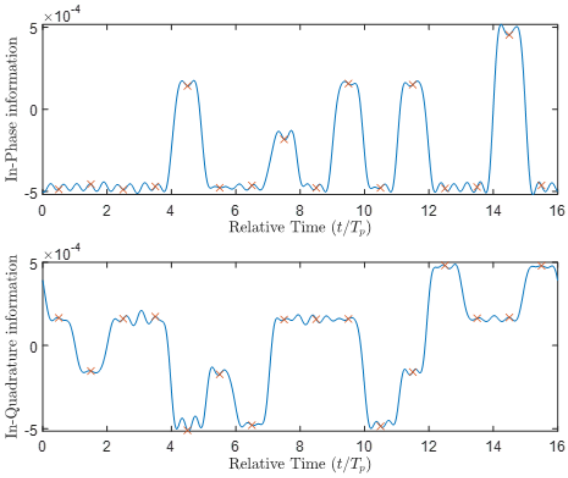

Fig. 14. Optical signal with only WDM interference.

A local oscillation around the signal level can be observed.

C. Signal distortion

We now enter the heart of the analysis of our system, first we will analyse the optical signal to appreciate the effect of the chromatic distortion at the end of the optical channel. Then we will use more advanced tools: eye diagrams. These representations allow us to evaluate the distortions introduced by the channel and then evaluate the quantity of errors introduced. The combined use with the constellation at the receiver will allow us to better understand the channel distortions. Finally, the error rate was calculated by comparing the input bit string with the one output to the receiver.

In Fig. 15 is reported the optical signal with the chromatic effect applied after 10 km. That signal comes from a 16-QAM scheme in order to better evaluate the distortion.

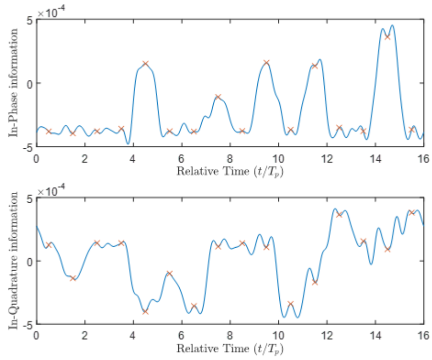

Fig. 15. Optical signal with cromatic dispersion.

In Fig. 16 is reported the eye diagram at the origin of an 64-QAM scheme, we can see that the structure is regular and symmetric, as we expect at the origin due to no distortion applied. Then in Fig. 17 and Fig. 18 we can see the eye diagram after 10 and 25 kilometres respectively.

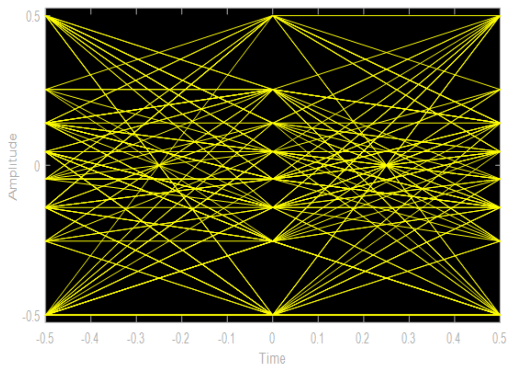

Fig. 16. Eye diagram of an 64-QAM signal at the origin.

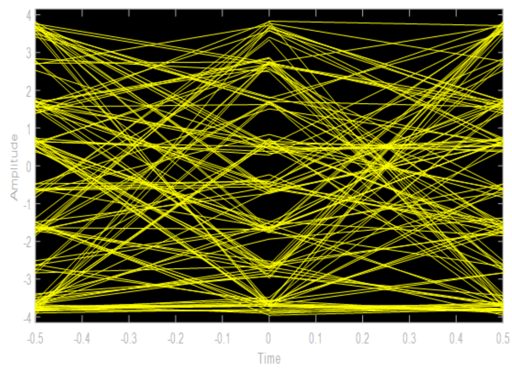

Fig. 17. Eye diagram of the signal after 10 Km.

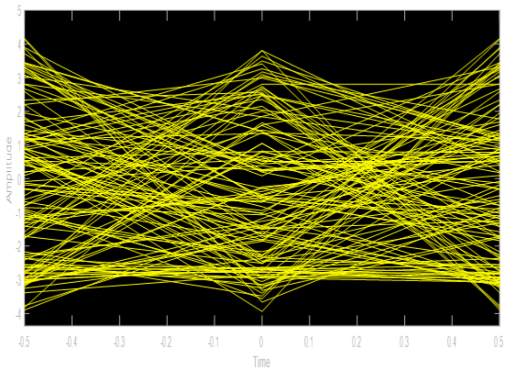

Fig. 18. Eye diagram of the signal after 25 Km.

We can see that just after 10 kilometres of propagation the eye diagram to show irregularities and the symmetry of the original structure has been broken, after 25 kilometres the structure is no more recognisable, indicating that the signal at the receiver has been significantly distorted.

This distortion is also reflected on the constellation, in Fig. 19 and Fig. 20 there are the constellations for the 10- and 25-kilometre range.

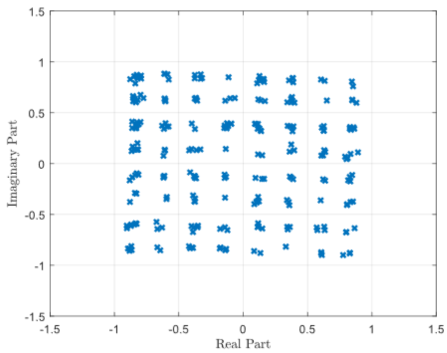

Fig. 19. Distortion of an 64-QAM constellation after 10 Km of propagation with CD.

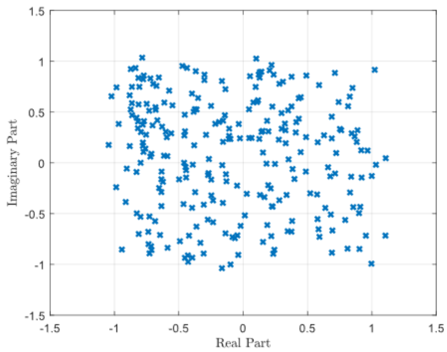

Fig. 20. Distortion of an 64-QAM constellation after 25 Km of propagation with CD.

After 10 kilometres of propagation, although the constellation has irregularities, the decision regions are still well defined. This is not the case after 25 kilometres of propagation, where the constellation structure is no longer clear. Error rate measurements were made, reported in TABLE I

Propagation	Error rate
10 Km	0.039%
25 Km	23.76%

TABLE I
BER MEASUREMENT

D. CD compensation

As we mentioned in the paragraph ... the Chromatic Dispersion is perfectly compensate by both optical and electrical means, in our system we have adopted the former route by applying two 25-Km sections of optical fibre with opposing D coefficients. The constellation is shown in Fig. 21 and the eye diagram in Fig. 22.

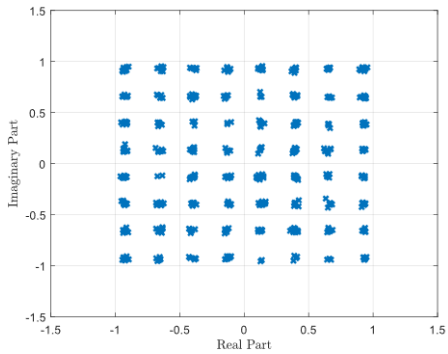

Fig. 21. Distortion of an 64-QAM constellation after applying CD compensation.

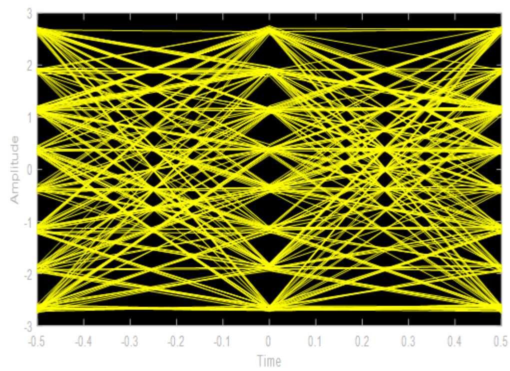

Fig. 22. Eye diagram after applying CD compensation.

As we can appreciate, both diagrams show minimal distortions, due solely to the effects of WDM (which cannot be compensated), in fact the measured error rate is $\sim 0.00\%$. This shows that CD is a propagation defect that can be perfectly compensated.

E. PRBS order

In the last part of our results, we want to compare the different orders of the PRBS sequence. We analysed the eye diagrams and constellations after a 15 Km propagation with CD applied. We have made the comparison between order 10 and order 50 and are shown in Fig.s 23,24,25,26.

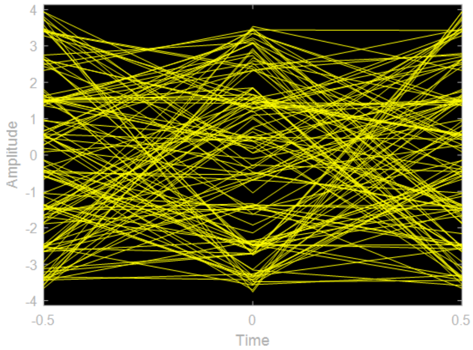

Fig. 23. Eye diagram with an PRBS order of 10 after 15 Km of propagation with CD.

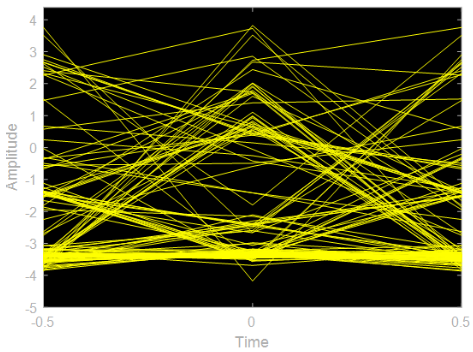

Fig. 24. Eye diagram with an PRBS order of 50 after 15 Km of propagation with CD.

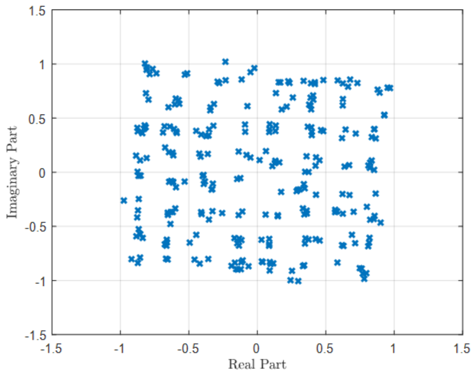

Fig. 25. Constellation of an 64-QAM with an PRBS order of 10 after 15 Km of propagation with CD.

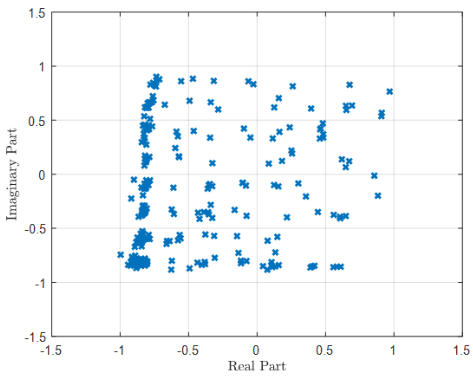

Fig. 26. Constellation of an 64-QAM with an PRBS order of 50 after 15 Km of propagation with CD.

The most noticeable effect as the order of the PRBS increases is that there is an accumulation of points in a certain area of the constellation and in the eye diagram.

VII. Conclusions

We have developed an complete communication systems that used the WDM technique to multiplex the signal of three channels into a single fiber. Using an I-Q-modulator with an 64-QAM and an Phase Diversity Coherent Receiver. We have provided the results of the simulations using an Pseudo Random Bit Sequence generator to generate our signal and later on we have studied the effects of the chromatic dispersion in the propagation, using constellations and eye diagrams.

The distortions produced by the Chromatic Dispersion can be perfectly compensated optically and electronically. Moreover in the absence of noise in the receiving and amplification devices, the signal arrives at its destination ideally without errors. The WDM multiplexing method introduces some interference due to the nature of the filters separating the various channels. On the contrary, the triple amount of data has been transmitted.

References

[1] Binh, L. N. (2014). *Optical fiber communication systems with matlab and simulink models.* Taylor & Francis Group.

[2] Michel C. Jeruchim, Philip Balaban, and K. Sam Shanmugan. 2000. *Simulation of Communication Systems: Modeling, Methodology and techniques* (2nd. ed.). Kluwer Academic Publishers, USA.

[3] Joan M. Gené Bernaus, 2022. *Fiber optic communication: optical fibers.* Optical communications group, UPC.

[4] Joan M. Gené Bernaus, 2022. *Fiber optic communication: external modulator.* Optical communications group, UPC.

[5] Joan M. Gené Bernaus, 2022. *Fiber optic communication: coherent detection.* Optical communications group, UPC.

[6] *PRBS Input Signals*. MathWorks Documentation, R2024a.

[7] Sheldon, Dec 22, 2023. *Demystifying Multiplexing Technologies: WDM, TDM and SDM*. Knowledge Center, FS Community.

[8] M. Bakni, G. James. 2005 *The IEEE wireless dictionary*. New York : Standards Information Network, IEEE Press.

[9] Qasim Chaudhari. 2020 *Pulse Shaping Filter*. Wireless Pi 2024.

Appendix A
Matlab code

In this appendix we have listed the most relevant parts of the MATLAB code. Starting with the generation of the PRBS string and the 64-QAM levels

```
Levels = 4;
SYMB = acos(linspace(1,-1,Levels))/pi;
mssg0 = SYMB(PseudoRandom(order,N_p));

b_I0 = mssg0(1,:);     % Bit sequence I
b_Q0 = mssg0(2,:);     % Bit sequence Q
```

Here is described how we obtain the levels from a PRBS using also a Serial-to-Parallel converter.

```
function [outputArg1] = PseudoRandom(
    order, length)
% PSEUDO RANDOM SEQUENCE
bitSeqI-Q = prbs(order, length * 3 * 2);
bitSeqI = bitSeq(1:length*3);
bitSeqQ = bitSeq(length*3+1:end);
for i = 1:length
bitTripleI = bitSeqI(3*i-2:3*i);
decValI = bitTripleI(1) * 4 + bitTripleI
    (2) * 2 + bitTripleI(3);
outputArg1(1,i) = decValI + 1;
% [...] Same for Q sequence
end
end
```

Here is described how the signal is generated for each channels

```
P_in = 1e-3; % Carrier power (W)
w1 = w0 - 5e10; % -50 GHz displacement
% Laser carrier
E_in1 = sqrt(P_in) * exp(1i*w1*2*pi*t);
% IQ Modulator output
E_out1 = IQmodulator(b_I1,b_Q1,E_in1,t,
    N_p,T_p,T_s,V_pi);
```

Here we described the behaviour of the optical filter.

```
function [OUT,A] = RectFilter(IN,
    filter_width,f_c,f)
A = fftshift(fft(fftshift(IN)));

mask = (f > f_c + filter_width / 2) | (f
    < f_c - filter_width / 2);

               A(mask) = 0;

OUT = fftshift(ifft(fftshift(A,2),[],2)
    ,2);
end
```

Here is described the Phase-Diversity coherent receiver.

```
E_out; Phase_out = angle(E_out);

R = 0.95;
P_in = 1e-3; % Local laser power (1mW)
Ii = R * abs(E_out) .* abs(sqrt(P_in)) .*
     cos(Phase_out);
I-Q = R * abs(E_out) .* abs(sqrt(P_in))
    .* sin(Phase_out);

a = 1/max(max(I-Q),max(Ii));
Ii = Ii * a;
I-Q = I-Q * a;
```

Then finally is described the behaviour of the Parallel-to-Serial converter.

```
function bitSeq = levelsToBinary(IN,
    numLevels)
    N = length(IN);
    levelStep = (maxValue - minValue) / (
        numLevels - 1);
    for i = 1:N
        level = round((IN(i) - minValue)
            / levelStep);

        binStr = dec2bin(level, ceil(log2
            (numLevels)));

        bitArray = binStr - '0';

        bitSeq(ceil(log2(numLevels)) * (i
            -1) + 1 : ceil(log2(numLevels)
            ) * i) = bitArray;
    end
end
```